Use of a gyrotropic birefringent plate as a quarter-wave plate

M. I. Shribak

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The polarization separation of the direct light beam and the light beam reflected from a mirror by means of a quarter-wave plate, exhibiting gyrotropic properties, is examined and the limitations imposed on it due to the gyrotropic properties are determined. The relationships obtained are used for a quartz plate, cut perpendicularly to the optical axis.

Optical reproduction devices having reflecting information carriers, such as laser, acoustic and video recorders,[1] have recently come into widespread use. The separation of the reading laser beam, incident on the information carrier, and the reflected modulated beam in these devices is accomplished by a polarization method using a quarter-wave plate. An inexpensive quarter-wave plate is necessary for the mass production of information recording systems. Such a plate is one made from quartz, transilluminated by an oblique light beam. The optical properties of this plate are examined in this paper. The possibilities of using it in different spectral regions for the polarization separation of the direct and reflected beams and also in circular polarizers and as a compensator are analyzed. The tolerances on the accuracy with which this plate is positioned and on the beam aperture are determined.

The Jones matrix method[2] was used for the calculation. The matrix for rotation of a coordinate system by the angle Ψ is denoted by $S(\Psi)$:

$$S(\psi)=\begin{pmatrix}\cos\psi & \sin\psi\\ -\sin\psi & \cos\psi\end{pmatrix},$$

the reflected light matrix is denoted by R:

$$R=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},$$

and the matrix of the gyrotropic birefringent plate is $M(\delta, \sigma)$:

$$M(\delta,\sigma)=\begin{pmatrix}\dfrac{e^{i\delta}+\sigma^2}{1+\sigma^2}; & -\dfrac{i\sigma(e^{i\delta}-1)}{1+\sigma^2}\\ \dfrac{i\sigma(e^{i\delta}-1)}{1+\sigma^2}; & \dfrac{1+e^{i\delta}\sigma^2}{1+\sigma^2}\end{pmatrix},$$

where σ is the ellipticity of the natural waves in the plate and δ is the phase shift between these waves. For a plane-parallel plate, cut from a gyrotropic uniaxial crystal perpendicularly to the optical axis, δ and σ are related to the incidence angle φ by the following expressions:

$$\delta=\frac{2\pi}{\lambda}dn_0\frac{\sqrt{\left(\dfrac{n_e^2-n_0^2}{2n_e^2n_0^2}\sin^2\varphi\right)^2+\left(n_0^2G_{33}+(G_{11}-G_{33})\sin^2\varphi\right)^2}}{\sqrt{1-\dfrac{n_0^2+n_e^2}{2n_e^2n_0^2}\sin^2\varphi}},$$

$$\sigma=\frac{G_{33}n_0^2+(G_{11}-G_{33})\sin^2\varphi}{\sqrt{\left(\dfrac{1}{2}\dfrac{n_e^2-n_0^2}{n_e^2n_0^2}\sin^2\varphi\right)^2+\left(G_{33}n_0^2+(G_{11}-G_{33})\sin^2\varphi\right)^2}-\dfrac{1}{2}\dfrac{n_e^2-n_0^2}{n_e^2n_0^2}\sin^2\varphi},$$

where λ is the wavelength of light in vacuum, d is the plate thickness, n_0 and n_e are the ordinary and extraordinary refractive indices, G_{11} and G_{33} are the components of the gyration tensor in the crystallophysical coordinate system.

Polarization separation of the reading beam and reflected beam, modulated by the information, is accomplished in optical information reproduction systems by means of a system comprising a polarization lightsplitter and a quarter-wave plate. As the quarter-wave plate, we use a plate cut from a gyrotropic uniaxial crystal perpendicularly to the optical axis. Let us consider the change in the polarization of a beam as it passes through the polarization lightsplitter, gyrotropic birefringent plate, mirror and is returned through this same plate. The incidence angle of the beam on the plate is equal to φ. The beam is incident perpendicularly on the mirror. The X axis is parallel to the polarization plane of the light, transmitted by the polarization lightsplitter, while the Y axis is perpendicular to it. The angle between the incidence plane of the beam on the plate and the X axis is Ψ (the polarization azimuth). The Jones vector $\begin{pmatrix}E_x\\ E_y\end{pmatrix}$ of the beam after passage through this system is written in the following form:

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$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = S(\Psi)\,M(\delta,\sigma)\,S(-\Psi)\,RS(-\Psi)\,M(\delta,\sigma)\,S(\Psi)\begin{pmatrix} E \\ 0 \end{pmatrix},$$

where $\begin{pmatrix} E \\ 0 \end{pmatrix}$ is the original Jones vector of the beam. After a multiplication of the matrices, we find that the efficiency η of the polarization is equal to:

$$\eta = \frac{E_y(E_y)^*}{EE^*} = 4\left(\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\right)^2 \times \left[1-\left(\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\right)^2\right]\sin^2$$
$$\times\left[2\Psi + \arcsin\left(\frac{2\sigma\sin\frac{\delta}{2}}{(1+\sigma^2)\sqrt{1-\left(\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\right)^2}}\right)\right]. \quad (1)$$

In order to analyze the noise of optical information reproduction devices[3] we need to have the degree of amplitude separation k, defined as

$$k = \sqrt{\frac{E_y(E_y)^*}{E_x(E_x)^*}} = \sqrt{\frac{\eta}{1-\eta}}. \quad (2)$$

Figure 1 shows graphs of the dependence of different k values (5, 10, 20, 40, 80, 160) on φ and Ψ, calculated for a quartz plate with a thickness $d = 1.39$ mm and a wavelength $\lambda = 0.6328\,\mu$m. The refractive indices of the quartz are taken from Ref. 4 and the coefficients G_{11} and G_{33} from Refs. 5, 6. For this plate the separation will be complete ($k = \infty$ or $\eta = 1$) for $\varphi = 10.17°$ and $\Psi = 28.35°$. The degree of amplitude separation will be at least 20 for a deviation of no more than $\pm 0.15°$ for the incidence angle φ from the optimum value and no more than $\pm 1.5°$ for the polarization azimuth Ψ; this corresponds to an energy efficiency $\eta = 0.998$ for the separation. Based on this, one can evaluate the tolerance on the accuracy with which the incidence angle φ and the polarization azimuth Ψ are adjusted.

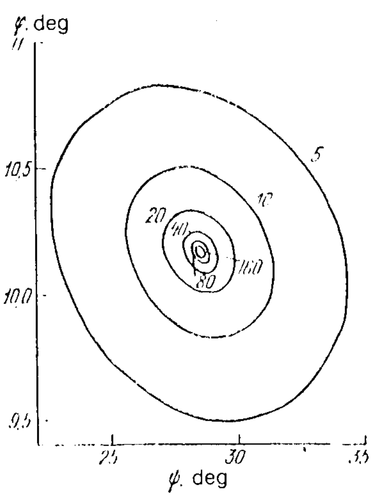

FIG. 1. Dependence of the degree of amplitude separation k on the incidence angle φ and polarization Ψ ($k = 5, 10, 20, 40, 80, 160$; $d = 1.39$ mm; $\lambda = 0.6328\,\mu$m).

Let us consider the polarization separation of uncollimated beams. We assume that the axial ray of the beam is incident on the plate at the optimum angle φ. Then an off-axis ray, striking the plate at the angle $\varphi + \omega$, will strike the plate at the angle $\varphi - \omega$ after reflection. Thus, the increase of δ and decrease of σ during the first passage through the plate will be partially compensated by the decrease of δ and the increase of σ during the return passage through the plate. This leads to a larger tolerance on the beam aperture than on the accuracy with which the axial ray is adjusted. The Jones vector of the off-axis ray $\begin{pmatrix} E_x \\ E_y \end{pmatrix}$ after passage through the system is written in the form

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = S(\Psi)\,M(\delta_2,\sigma_2)\,S(-\Psi)\,RS(-\Psi)\,M(\delta_1,\sigma_1)\,S(\Psi)\begin{pmatrix} E \\ 0 \end{pmatrix},$$

where $\begin{pmatrix} E \\ 0 \end{pmatrix}$ is the Jones vector of the original ray, δ_1 and σ_1 are the values of the phase shift and ellipticity for the incidence angle $\varphi + \omega$, and δ_2 and σ_2 are the values of the phase shift and ellipticity for the incidence angle $\varphi - \omega$. After a multiplication of the matrices we find that the efficiency η of the polarization separation is equal to:

$$\eta = \frac{E_y(E_y)^*}{EE^*} = (a_1c_2 - a_2c_1)^2 + [(a_1b_2 + a_2b_1)\sin 2\Psi - (b_1c_2 + b_2c_1)\cos 2\Psi]^2, \quad (3)$$

where

$$a_j = \cos\frac{\delta_j}{2}, \quad b_j = \frac{1-\sigma_j^2}{1+\sigma_j^2}\sin\frac{\delta_j}{2},$$
$$c_j = \frac{2\sigma_j}{1+\sigma_j^2}\sin\frac{\delta_j}{2}, \quad j = 1, 2.$$

Figure 2 shows a graph of the dependence on ω of the degree of amplitude separation k, related to η by Eq. (2) ($d = 1.39$ mm, $\lambda = 0.6328\,\mu$m and $\Psi = 28.35°$). As seen from this graph, a beam aperture of $\pm 2°$ is permissible for this plate when it is positioned precisely since in this case the degree of amplitude separation of the beam is greater than 20.

It follows from Eq. (1) that the separation will be complete ($\eta = 1$) for

$$\begin{cases} \left(\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\right)^2 = \frac{1}{2} \\ 2\Psi = \pm 90° - \arcsin\left(\frac{2\sigma}{1-\sigma^2}\right). \end{cases} \quad (4)$$

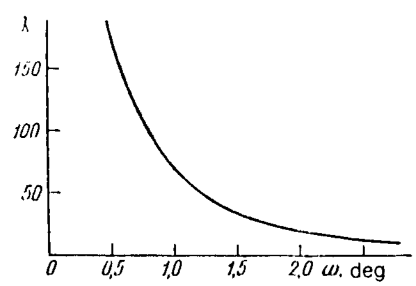

FIG. 2. Dependence of the degree of amplitude separation k and beam deflection ω on the optimum incidence angle φ ($d = 1.39$ mm, $\lambda = 0.6328\,\mu$m, $\Psi = 28.35°$, $\varphi = 10.17°$).

When these conditions are satisfied, the plate will operate as a quarter-wave plate. From Eq. (4) we have the following limiting condition on σ: $\sigma < \sqrt{2} - 1 \approx 0.41$.

Since σ is determined by the value of the incidence angle of the light on the plate, a limitation on this angle arises, setting a lower limit on its value. Thus, for quartz the minimum incidence angle is equal to 7.5° ($\lambda = 0.6328\,\mu$m). This is a fundamental distinction between the optical properties of gyrotropic and nongyrotropic quarter-wave plates. In the latter (since $\sigma = 0$) a quarter-wave plate of appropriate thickness can be obtained for any incidence angle except $\varphi = 0$.

Let us consider the optical properties of a gyrotropic birefringent plate for a single pass of the beam. Let us assume the coordinate system is rotated by the angle α with respect to the original system. Then in this coordinate system the Jones matrix $\widetilde{M}$ of the plate will have the form:

$$\widetilde{M} = S(-\alpha)\, S(-\Psi)\, M(\delta, \sigma)\, S(\Psi)\, S(\alpha)$$

$$= \begin{pmatrix} \cos\frac{\delta}{2} + i\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\cos 2(\Psi+\alpha); \\ -\frac{2\delta}{1+\sigma^2}\sin\frac{\delta}{2} + i\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\sin 2(\Psi+\alpha) \\ \frac{2\sigma}{1+\sigma^2}\sin\frac{\delta}{2} + i\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\sin 2(\Psi+\alpha); \\ \cos\frac{\delta}{2} - i\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\cos 2(\Psi+\alpha) \end{pmatrix}. \qquad (5)$$

When conditions (4) are satisfied, the matrix (5) becomes:

$$\widetilde{M} = \frac{1}{\sqrt{2}}\begin{pmatrix} \cos 2(\alpha+\Psi) - i\sin 2\Psi; \\ \sin 2(\alpha+\Psi) - i\cos 2\Psi \\ \sin 2(\alpha+\psi) - i\cos 2\Psi; \\ -\cos 2(\alpha+\Psi) - i\sin 2\Psi \end{pmatrix}. \qquad (6)$$

If $\alpha = 0$, then

$$\widetilde{M} = \frac{1}{\sqrt{2}}\begin{pmatrix} \exp(-2i\Psi); & -i\exp(2i\Psi) \\ i\exp(-2i\Psi); & -\exp(2i\psi) \end{pmatrix}.$$

As is seen, in this case the plate converts linearly polarized light into circularly polarized, i.e., it acts like a circular polarizer:

$$\widetilde{M}\begin{pmatrix} E \\ 0 \end{pmatrix} = \frac{E}{\sqrt{2}}\exp(-2i\Psi)\begin{pmatrix} 1 \\ i \end{pmatrix},$$

where $\begin{pmatrix} E \\ 0 \end{pmatrix}$ is the Jones vector of the light striking the plate and $(E/\sqrt{2})\exp(-2i\Psi)\begin{pmatrix} 1 \\ i \end{pmatrix}$ is the Jones vector of the light emerging from the plate.

Let us determine the restrictions on the accuracy with which this plate is positioned and on the beam aperture. After substituting the value $\alpha = 0$ into the matrix (5), we find that this plate converts light having the original Jones vector $\begin{pmatrix} E \\ 0 \end{pmatrix}$ into light having the Jones vector $\begin{pmatrix} E_x \\ E_y \end{pmatrix}$, where

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \cos\frac{\delta}{2} + i\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\cos 2\Psi \\ \frac{2\sigma}{1+\sigma^2}\sin\frac{\delta}{2} + i\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\sin 2\Psi \end{pmatrix} E.$$

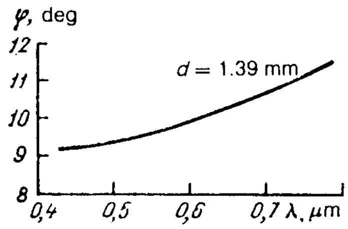

FIG. 3. Dependence of the optimum incidence angle φ on the light wavelength λ ($d = 1.39$ mm).

The ellipticity γ of this light[2] will be determined from the formula

$$\sin 2\gamma = 2\left(\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\right)\sqrt{1 - \left(\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\right)^2}$$

$$\times \sin\left(2\Psi + \arcsin\frac{2\sigma\sin\frac{\delta}{2}}{(1+\sigma^2)\sqrt{1-\left(\frac{1-\sigma^2}{1+\sigma^2}\sin\frac{\delta}{2}\right)^2}}\right).$$

As is seen, $\sin^2 2\gamma$ is equal to the efficiency η of the polarization separation of the beam (1). Therefore,

$$\operatorname{tg} 2\gamma = \sqrt{\frac{\eta}{1-\eta}} = k.$$

A deviation of the ellipticity from 45° by 2° corresponds to the line $k = 14$ in Fig. 1. Thus, in order that the ellipticity of the light after passage through the plate be equal to 45° ± 2°, the accuracy of the beam incidence must be ± 0.30°. Then we can determine the tolerance on the beam aperture also.

If $\alpha = 45°$, the matrix (6) becomes

$$\widetilde{M} = \frac{1}{\sqrt{2}}\begin{pmatrix} i & \sin 2\Psi & \cos 2\Psi \\ -i & \cos 2\Psi & \sin 2\Psi \end{pmatrix}.$$

A plate with such a Jones matrix can be used as a compensator. In this case the plate converts elliptically polarized light having the Jones vector $\begin{pmatrix} A \\ iB \end{pmatrix}$ into linearly polarized. The angle between the polarization plane of the transmitted light and the polarization plane of component A of the incident light is χ and it will be equal to:

$$\chi = 90° - 2\Psi + \operatorname{arctg}\frac{B}{A} = \arcsin\frac{2\sigma}{\sqrt{1-\sigma^2}} + \operatorname{arctg}\frac{B}{A}.$$

In this case, in contrast to the usual compensator, we have the addition term $\arcsin(2\sigma/\sqrt{1-\sigma^2})$, attributable to the gyrotropic properties.

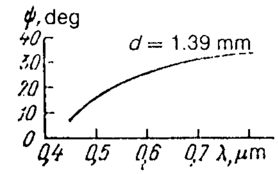

FIG. 4. Dependence of the optimum polarization azimuth ψ on the light wavelength λ ($d = 1.39$ mm).

The optical properties of the plate are determined by using Eqs. (4). The Eqs. (4) make it possible to calculate the required values of Ψ and φ for the entire transmission spectrum of the material. Figures 3 and 4 show the dependence of the incidence angle φ and the polarization azimuth Ψ on the wavelength λ for a quartz plate having a thickness $d = 1.39$ mm. This method can also be used to examine plates of different thicknesses and of different materials.

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Statistical analysis of the position of the axis of the natural waves of an optical resonator

E. F. Ishchenko and A. V. Karpilenko

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The concept of the statistical axial caustic (SAC), characterizing the rms deviation of the axis, is introduced on the basis of an analysis of the stability of the axis of an optical resonator. The basic characteristics of the SAC, making it possible to match the operating focusing cross section to the resonator parameters, are discussed. The laws for the transformation of the SAC by external stable and unstable optical systems are examined.

INTRODUCTION

Lasers have now come into widespread use in various instruments and devices, the operation of which is based on a determination of a certain direction in space. Usually, this direction is linked to the axis of the natural waves of the optical resonator or, in other words, to the axis of the directivity pattern (ADP) of the laser radiation, as a result of which it is very important to investigate the spatial stability of the ADP. A number of papers[1–3] have been devoted to this question, in which the variation of the position of the ADP is considered as a consequence of deterministic misadjustments and perturbations of the optical elements of the resonator. However, our representations of the factors influencing the variation of the optical structure of the resonator often do not permit the position of the ADP to be determined uniquely. If the principal regular factors are excluded, then the remaining displacements of the ADP are often random in character. This fact leads to the idea of using statistical analysis methods for considering the stability of the ADP.

CONCEPT OF THE STATISTICAL AXIAL CAUSTIC

Let us consider an arbitrary resonator formed by a series of optical elements. These elements can include mirrors, prisms, diaphragms, the optical media filling the resonator (including the active medium also) and other devices.

The position of the ADP is determined by the arrangement of the refracting and reflecting surfaces as well as the optical density distribution of the media. The initial set of these characteristics determines the nominal structure of the resonator. Let us assume a random perturbation of the ith parameter causes a proportional displacement of the axis of the resonator beam in some cross section (z) outside the resonator. Then the proportionality coefficient can be considered as some transfer coefficient $c_i(z)$, determining the significance of the perturbing factor. We will also specify the statistical interrelationship of the different perturbations by the matrix of moments λ_{ij}. When all perturbations are taken into consideration, the variance of the axis displacement in the cross section (z) being considered will be defined as[4]

$$\sigma^2(z) = \sum_{i,j} \lambda_{ij} c_i(z) c_j(z), \tag{1}$$

where λ_{ij} are the second-order central moments of the different perturbations $(\lambda_{ij} = \lambda_{ji})$.

As long as the misadjustment perturbations of the resonator are so small that the ADP coincides with the line along which a ray propagates in a self-conjugate manner after each diversion, the $c_i(z)$ dependence is linear:

$$c_i(z) = c_{i0} + c_{i1} z. \tag{2}$$

The coefficients of the $c_i(z)$ dependence are determined by

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